Complete Solutions Manual
to Accompany

Elementary Linear Algebra

EIGHTH EDITION

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CHAPTER 1
Systems of Linear Equations

Section 1.1 Introduction to Systems of Linear Equations

2. Because the term xy cannot be rewritten as $ax + by$ for any real numbers a and b, the equation cannot be written in the form $a_1x + a_2y = b$. So, this equation is *not* linear in the variables x and y.

4. Because the terms x^2 and y^2 cannot be rewritten as $ax + by$ for any real numbers a and b, the equation cannot be written in the form $a_1x + a_2y = b$. So, this equation is *not* linear in the variables x and y.

6. Because the equation is in the form $a_1x + a_2y = b$, it is linear in the variables x and y.

8. Choosing y as the free variable, let $y = t$ and obtain

$$3x - \tfrac{1}{2}t = 9$$
$$3x = 9 + \tfrac{1}{2}t$$
$$x = 3 + \tfrac{1}{6}t.$$

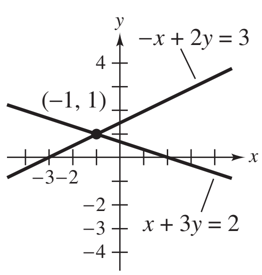
So, you can describe the solution set as $x = 3 + \frac{1}{6}t$ and $y = t$, where t is any real number.

10. Choosing x_2 and x_3 as free variables, let $x_2 = s$ and $x_3 = t$ and obtain $12x_1 + 24s - 36t = 12$.

$$x_1 + 2s - 3t = 1$$
$$x_1 = 1 - 2s + 3t.$$

So, you can describe the solution set as $x_1 = 1 - 2s + 3t$, $x_3 = t$, and $x_2 = s$, where s and t are any real number.

12.

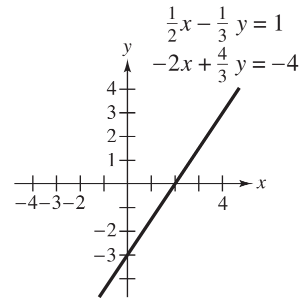


$$x + 3y = 2$$
$$-x + 2y = 3$$

Adding the first equation to the second equation produces a new second equation, $5y = 5$ or $y = 1$.

So, $x = 2 - 3y = 2 - 3(1)$, and the solution is: $x = -1$, $y = 1$. This is the point where the two lines intersect.

14.



The two lines coincide.

Multiplying the first equation by 2 produces a new first equation.

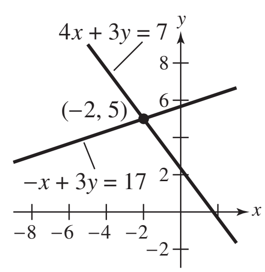
$$x - \tfrac{2}{3}y = 2$$
$$-2x + \tfrac{4}{3}y = -4$$

Adding 2 times the first equation to the second equation produces a new second equation.

$$x - \tfrac{2}{3}y = 2$$
$$0 = 0$$

Choosing $y = t$ as the free variable, you obtain $x = \frac{2}{3}t + 2$. So, you can describe the solution set as $x = \frac{2}{3}t + 2$ and $y = t$, where t is any real number.

16.

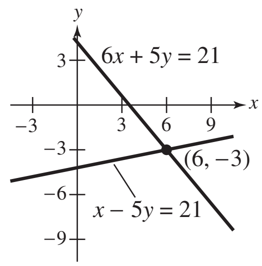


$$-x + 3y = 17$$
$$4x + 3y = 7$$

Subtracting the first equation from the second equation produces a new second equation, $5x = -10$ or $x = -2$.

So, $4(-2) + 3y = 7$ or $y = 5$, and the solution is: $x = -2$, $y = 5$. This is the point where the two lines intersect.

18.

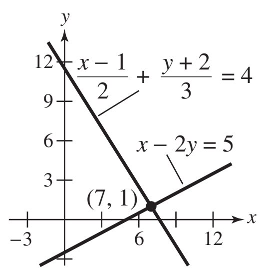


$$x - 5y = 21$$
$$6x + 5y = 21$$

Adding the first equation to the second equation produces a new second equation, $7x = 42$ or $x = 6$.

So, $6 - 5y = 21$ or $y = -3$, and the solution is: $x = 6$, $y = -3$. This is the point where the two lines intersect.

20.



$$\frac{x-1}{2} + \frac{y+2}{3} = 4$$
$$x - 2y = 5$$

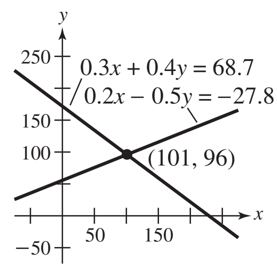
Multiplying the first equation by 6 produces a new first equation.

$$3x + 2y = 23$$
$$x - 2y = 5$$

Adding the first equation to the second equation produces a new second equation, $4x = 28$ or $x = 7$.

So, $7 - 2y = 5$ or $y = 1$, and the solution is: $x = 7$, $y = 1$. This is the point where the two lines intersect.

22.



$$0.2x - 0.5y = -27.8$$
$$0.3x + 0.4y = 68.7$$

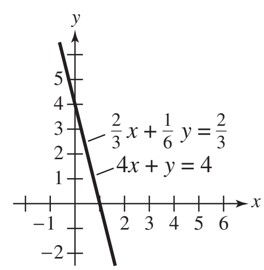
Multiplying the first equation by 40 and the second equation by 50 produces new equations.

$$8x - 20y = -1112$$
$$15x + 20y = 3435$$

Adding the first equation to the second equation produces a new second equation, $23x = 2323$ or $x = 101$.

So, $8(101) - 20y = -1112$ or $y = 96$, and the solution is: $x = 101$, $y = 96$. This is the point where the two lines intersect.

24.



$$\tfrac{2}{3}x + \tfrac{1}{6}y = \tfrac{2}{3}$$
$$4x + y = 4$$

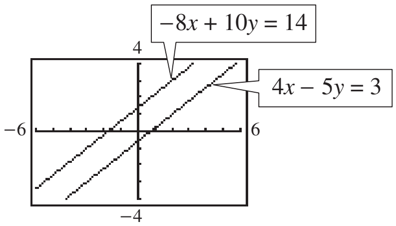
Adding 6 times the first equation to the second equation produces a new second equation, $0 = 0$. Choosing $x = t$ as the free variable, you obtain $y = 4 - 4t$. So, you can describe the solution as $x = t$ and $y = 4 - 4t$, where t is any real number.

26. From Equation 2 you have $x_2 = 3$. Substituting this value into Equation 1 produces $2x_1 - 12 = 6$ or $x_1 = 9$. So, the system has exactly one solution: $x_1 = 9$ and $x_2 = 3$.

28. From Equation 3 you have $z = 2$. Substituting this value into Equation 2 produces $3y + 2 = 11$ or $y = 3$. Finally, substituting $y = 3$ into Equation 1, you obtain $x - 3 = 5$ or $x = 8$. So, the system has exactly one solution: $x = 8$, $y = 3$, and $z = 2$.

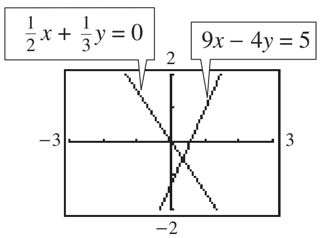
30. From the second equation you have $x_2 = 0$. Substituting this value into Equation 1 produces $x_1 + x_3 = 0$. Choosing x_3 as the free variable, you have $x_3 = t$ and obtain $x_1 + t = 0$ or $x_1 = -t$. So, you can describe the solution set as $x_1 = -t$, $x_2 = 0$, and $x_3 = t$.

32. (a)



(b) This system is inconsistent, because you see two parallel lines on the graph of the system.

34. (a)



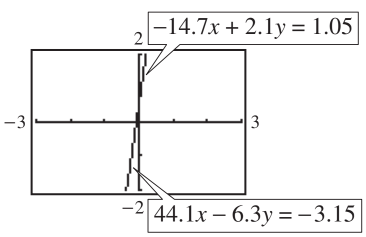
(b) Two lines corresponding to two equations intersect at a point, so this system is consistent.

(c) The solution is approximately $x = \frac{1}{3}$ and $y = -\frac{1}{2}$.

(d) Adding -18 times the second equation to the first equation, you obtain $-10y = 5$ or $y = -\frac{1}{2}$. Substituting $y = -\frac{1}{2}$ into the first equation, you obtain $9x = 3$ or $x = \frac{1}{3}$. The solution is: $x = \frac{1}{3}$ and $y = -\frac{1}{2}$.

(e) The solutions in (c) and (d) are the same.

36. (a)



(b) Because the lines coincide, the system is consistent.

(c) All solutions of this system lie on the line $y = 7x + \frac{1}{2}$. So, let $x = t$, then the solution set is $x = t, y = 7t + \frac{1}{2}$, where t is any real number.

(d) Adding 3 times the first equation to the second equation you obtain

$$\begin{aligned} -44.1x + 6.3y &= 3.15 \\ 0 &= 0. \end{aligned}$$

Choosing $x = t$ as a free variable, you obtain $-14.7t + 2.1y = 1.05$ or $-147t + 21y = 105$ or $y = 7t + \frac{1}{2}$.

So, you can describe the solution set as $x = t, y = 7t + \frac{1}{2}$, where t is any real number.

(e) The solutions in (c) and (d) are the same.

38. Adding -2 times the first equation to the second equation produces a new second equation.

$$\begin{aligned} 3x + 2y &= 2 \\ 0 &= 10 \end{aligned}$$

Because the second equation is a false statement, the original system of equations has no solution.

40. Adding -6 times the first equation to the second equation produces a new second equation.

$$\begin{aligned} x_1 - 2x_2 &= 0 \\ 14x_2 &= 0 \end{aligned}$$

Now, using back-substitution, the system has exactly one solution: $x_1 = 0$ and $x_2 = 0$.

42. Multiplying the first equation by $\frac{3}{2}$ produces a new first equation.

$$\begin{aligned} x_1 + \tfrac{1}{4}x_2 &= 0 \\ 4x_1 + x_2 &= 0 \end{aligned}$$

Adding -4 times the first equation to the second equation produces a new second equation.

$$\begin{aligned} x_1 + \tfrac{1}{4}x_2 &= 0 \\ 0 &= 0 \end{aligned}$$

Choosing $x_2 = t$ as the free variable, you obtain $x_1 = -\frac{1}{4}t$. So you can describe the solution set as $x_1 = -\frac{1}{4}t$ and $x_2 = t$, where t is any real number.

44. To begin, change the form of the first equation.

$$\begin{aligned} \frac{x_1}{3} + \frac{x_2}{2} &= -\frac{5}{6} \\ 3x_1 - x_2 &= -2 \end{aligned}$$

Multiplying the first equation by 3 yields a new first equation.

$$\begin{aligned} x_1 + \frac{3}{2}x_2 &= -\frac{5}{2} \\ 3x_1 - x_2 &= -2 \end{aligned}$$

Adding –3 times the first equation to the second equation produces a new second equation.

$$\begin{aligned} x_1 + \frac{3}{2}x_2 &= -\frac{5}{2} \\ -\frac{11}{2}x_2 &= \frac{11}{2} \end{aligned}$$

Multiplying the second equation by $-\dfrac{2}{11}$ yields a new second equation.

$$\begin{aligned} x_1 + \frac{3}{2}x_2 &= -\frac{5}{2} \\ x_2 &= -1 \end{aligned}$$

Now, using back-substitution, the system has exactly one solution: $x_1 = -1$ and $x_2 = -1$.

46. Multiplying the first equation by 20 and the second equation by 100 produces a new system.

$$\begin{aligned} x_1 - 0.6x_2 &= 4.2 \\ 7x_1 + 2x_2 &= 17 \end{aligned}$$

Adding -7 times the first equation to the second equation produces a new second equation.

$$\begin{aligned} x_1 - 0.6x_2 &= 4.2 \\ 6.2x_2 &= -12.4 \end{aligned}$$

Now, using back-substitution, the system has exactly one solution: $x_1 = 3$ and $x_2 = -2$.

48. Adding the first equation to the second equation yields a new second equation.

$$\begin{aligned} x + y + z &= 2 \\ 4y + 3z &= 10 \\ 4x + y &= 4 \end{aligned}$$

Adding -4 times the first equation to the third equation yields a new third equation.

$$\begin{aligned} x + y + z &= 2 \\ 4y + 3z &= 10 \\ -3y - 4z &= -4 \end{aligned}$$

Dividing the second equation by 4 yields a new second equation.

$$\begin{aligned} x + y + z &= 2 \\ y + \tfrac{3}{4}z &= \tfrac{5}{2} \\ -3y - 4z &= -4 \end{aligned}$$

Adding 3 times the second equation to the third equation yields a new third equation.

$$\begin{aligned} x + y + z &= 2 \\ y + \tfrac{3}{4}z &= \tfrac{5}{2} \\ -\tfrac{7}{4}z &= \tfrac{7}{2} \end{aligned}$$

Multiplying the third equation by $-\frac{4}{7}$ yields a new third equation.

$$\begin{aligned} x + y + z &= 2 \\ y + \tfrac{3}{4}z &= \tfrac{5}{2} \\ z &= -2 \end{aligned}$$

Now, using back-substitution the system has exactly one solution: $x = 0$, $y = 4$, and $z = -2$.

50. Interchanging the first and third equations yields a new system.

$$\begin{aligned} x_1 - 11x_2 + 4x_3 &= 3 \\ 2x_1 + 4x_2 - x_3 &= 7 \\ 5x_1 - 3x_2 + 2x_3 &= 3 \end{aligned}$$

Adding -2 times the first equation to the second equation yields a new second equation.

$$\begin{aligned} x_1 - 11x_2 + 4x_3 &= 3 \\ 26x_2 - 9x_3 &= 1 \\ 5x_1 - 3x_2 + 2x_3 &= 3 \end{aligned}$$

Adding -5 times the first equation to the third equation yields a new third equation.

$$\begin{aligned} x_1 - 11x_2 + 4x_3 &= 3 \\ 26x_2 - 9x_3 &= 1 \\ 52x_2 - 18x_3 &= -12 \end{aligned}$$

At this point, you realize that Equations 2 and 3 cannot both be satisfied. So, the original system of equations has no solution.

52. Adding -4 times the first equation to the second equation and adding -2 times the first equation to the third equation produces new second and third equations.

$$\begin{aligned} x_1 + 4x_3 &= 13 \\ -2x_2 - 15x_3 &= -45 \\ -2x_2 - 15x_3 &= -45 \end{aligned}$$

The third equation can be disregarded because it is the same as the second one. Choosing x_3 as a free variable and letting $x_3 = t$, you can describe the solution as

$x_1 = 13 - 4t$

$x_2 = \frac{45}{2} - \frac{15}{2}t$

$x_3 = t$, where t is any real number.

54. Adding -3 times the first equation to the second equation produces a new second equation.

$$\begin{aligned} x_1 - 2x_2 + 5x_3 &= 2 \\ 8x_2 - 16x_3 &= -8 \end{aligned}$$

Dividing the second equation by 8 yields a new second equation.

$$\begin{aligned} x_1 - 2x_2 + 5x_3 &= 2 \\ x_2 - 2x_3 &= -1 \end{aligned}$$

Adding 2 times the second equation to the first equation yields a new first equation.

$$\begin{aligned} x_1 + x_3 &= 0 \\ x_2 - 2x_3 &= -1 \end{aligned}$$

Letting $x_3 = t$ be the free variable, you can describe the solution as $x_1 = -t$, $x_2 = 2t - 1$, and $x_3 = t$, where t is any real number.

56. Adding 3 times the first equation to the fourth equation yields

$$\begin{aligned} -x_1 \quad\quad\quad\quad + 2x_4 &= 1 \\ 4x_2 - x_3 - x_4 &= 2 \\ x_2 \quad\quad - x_4 &= 0 \\ -2x_2 + 3x_3 + 6x_4 &= 7. \end{aligned}$$

Interchanging the second equation with the third equation yields

$$\begin{aligned} -x_1 \quad\quad\quad\quad + 2x_4 &= 1 \\ x_2 \quad\quad - x_4 &= 0 \\ 4x_2 - x_3 - x_4 &= 2 \\ -2x_2 + 3x_3 + 6x_4 &= 7. \end{aligned}$$

Adding -4 times the second equation to the third equation, and adding -2 times the second equation to the fourth equation yields

$$\begin{aligned} -x_1 \quad\quad + 2x_4 &= 1 \\ x_2 \quad - x_4 &= 0 \\ -x_3 + 3x_4 &= 2 \\ 3x_3 + 4x_4 &= 7. \end{aligned}$$

Adding 3 times the second equation to the third equation yields

$$\begin{aligned} -x_1 \quad\quad + 2x_4 &= 1 \\ x_2 \quad - x_4 &= 0 \\ -x_3 + 3x_4 &= 2 \\ 13x_4 &= 13. \end{aligned}$$

Using back-substitution, the original system has exactly one solution: $x_1 = 1, x_2 = 1, x_3 = 1$, and $x_4 = 1$.

Answers may vary slightly for Exercises 58–62.

58. Using a software program or graphing utility, you obtain $x = 0.8,\ y = 1.2,\ z = -2.4$.

60. Using a software program or graphing utility, you obtain $x = 10, y = -20, z = 40, w = -12$.

62. Using a software program or graphing utility, you obtain $x = 6.8813,\ y = -163.3111,\ z = -210.2915,$ $w = -59.2913$.

64. $x = y = z = 0$ is clearly a solution.

Dividing the first equation by 2 produces

$$\begin{aligned} x + \tfrac{3}{2}y \quad\quad &= 0 \\ 4x + 3y - z &= 0 \\ 8x + 3y + 3z &= 0. \end{aligned}$$

Adding -4 times the first equation to the second equation, and -8 times the first equation to the third, yields

$$\begin{aligned} x + \tfrac{3}{2}y \quad\quad &= 0 \\ -3y - z &= 0 \\ -9y + 3z &= 0. \end{aligned}$$

Adding -3 times the second equation to the third equation yields

$$\begin{aligned} x + \tfrac{3}{2}y \quad\quad &= 0 \\ -3y - z &= 0 \\ 6z &= 0. \end{aligned}$$

Using back-substitution, you conclude there is exactly one solution: $x = y = z = 0$.

66. $x = y = z = 0$ is clearly a solution.

Dividing the second equation by 2 yields a new second equation.

$$\begin{aligned} 16x + 3y + z &= 0 \\ 8x + y - \tfrac{1}{2}z &= 0 \end{aligned}$$

Adding -3 times the second equation to the first equation produces a new first equation.

$$\begin{aligned} -8x \quad\quad + \tfrac{5}{2}z &= 0 \\ 8x + y - \tfrac{1}{2}z &= 0 \end{aligned}$$

Letting $z = t$ be the free variable, you can describe the solution as $x = \frac{5}{16}t, y = -2t$, and $z = t$, where t is any real number.

68. Let $x =$ the speed of the plane that leaves first and $y =$ the speed of the plane that leaves second.

$$\begin{aligned} y - x &= 80 && \text{Equation 1} \\ 2x + \tfrac{3}{2}y &= 3200 && \text{Equation 2} \end{aligned}$$

$$\begin{aligned} -2x + 2y &= 160 \\ 2x + \tfrac{3}{2}y &= 3200 \\ \hline \tfrac{7}{2}y &= 3360 \\ y &= 960 \end{aligned}$$

$$\begin{aligned} 960 - x &= 80 \\ x &= 880 \end{aligned}$$

Solution: First plane: 880 kilometers per hour; second plane: 960 kilometers per hour

70. (a) False. Any system of linear equations is either consistent, which means it has a unique solution, or infinitely many solutions; or inconsistent, which means it has no solution. This result is stated on page 5, and will be proved later in Theorem 2.5.

(b) True. See definition on page 6.

(c) False. Consider the following system of three linear equations with two variables.

$$\begin{aligned} 2x + y &= -3 \\ -6x - 3y &= 9 \\ x &= 1. \end{aligned}$$

The solution to this system is: $x = 1,\ y = -5$.

72. Because $x_1 = t$ and $x_2 = s$, you can write $x_3 = 3 + s - t = 3 + x_2 - x_1$. One system could be

$$\begin{aligned} x_1 - x_2 + x_3 &= 3 \\ -x_1 + x_2 - x_3 &= -3 \end{aligned}$$

Letting $x_3 = t$ and $x_2 = s$ be the free variables, you can describe the solution as $x_1 = 3 + s - t$, $x_2 = s$, and $x_3 = t$, where t and s are any real numbers.

74. Substituting $A = \dfrac{1}{x}$ and $B = \dfrac{1}{y}$ into the original system yields

$$\begin{aligned} 3A + 2B &= -1 \\ 2A - 3B &= -\frac{17}{6}. \end{aligned}$$

Reduce the system to row-echelon form.

$$\begin{aligned} 27A + 18B &= -9 \\ 12A - 18B &= -17 \end{aligned}$$

$$\begin{aligned} 27A + 18B &= -9 \\ 39A \qquad &= -26 \end{aligned}$$

Using back substitution, $A = -\dfrac{2}{3}$ and $B = \dfrac{1}{2}$. Because $A = \dfrac{1}{x}$ and $B = \dfrac{1}{y}$, the solution of the original system of equations is: $x = -\dfrac{3}{2}$ and $y = 2$.

76. Substituting $A = \dfrac{1}{x}$, $B = \dfrac{1}{y}$, and $C = \dfrac{1}{z}$ into the original system yields

$$\begin{aligned} 2A + B - 2C &= 5 \\ 3A - 4B \qquad &= -1. \\ 2A + B + 3C &= 0 \end{aligned}$$

Reduce the system to row-echelon form.

$$\begin{aligned} 2A + B - 2C &= 5 \\ 3A - 4B \qquad &= -1 \\ 5C &= -5 \end{aligned}$$

$$\begin{aligned} 3A - 4B \qquad &= -1 \\ -11B + 6C &= -17 \\ 5C &= -5 \end{aligned}$$

So, $C = -1$. Using back-substitution, $-11B + 6(-1) = -17$, or $B = 1$ and $3A - 4(1) = -1$, or $A = 1$. Because $A = 1/x$, $B = 1/y$, and $C = 1/z$, the solution of the original system of equations is: $x = 1$, $y = 1$, and $z = -1$.

78. Multiplying the first equation by $\sin\theta$ and the second by $\cos\theta$ produces

$$\begin{aligned} (\sin\theta\cos\theta)x + (\sin^2\theta)y &= \sin\theta \\ -(\sin\theta\cos\theta)x + (\cos^2\theta)y &= \cos\theta. \end{aligned}$$

Adding these two equations yields

$$\begin{aligned} (\sin^2\theta + \cos^2\theta)y &= \sin\theta + \cos\theta \\ y &= \sin\theta + \cos\theta. \end{aligned}$$

So, $(\cos\theta)x + (\sin\theta)y = (\cos\theta)x + \sin\theta(\sin\theta + \cos\theta) = 1$ and

$$x = \frac{(1 - \sin^2\theta - \sin\theta\cos\theta)}{\cos\theta} = \frac{(\cos^2\theta - \sin\theta\cos\theta)}{\cos\theta} = \cos\theta - \sin\theta.$$

Finally, the solution is $x = \cos\theta - \sin\theta$ and $y = \cos\theta + \sin\theta$.

80. Reduce the system to row-echelon form.

$$x + ky = 0$$
$$(1 - k^2)y = 0$$

$$x + ky = 0$$
$$y = 0,\ 1 - k^2 \neq 0$$

$$x = 0$$
$$y = 0,\ 1 - k^2 \neq 0$$

If $1 - k^2 \neq 0$, that is if $k \neq \pm 1$, the system will have exactly one solution.

82. Reduce the system to row-echelon form.

$$x + 2y + kz = 6$$
$$(8 - 3k)z = -14$$

This system will have no solution if $8 - 3k = 0$, that is, $k = \frac{8}{3}$.

84. Reduce the system to row-echelon form.

$$kx + y = 16$$
$$(4k + 3)x = 0$$

The system will have an infinite number of solutions when $4k + 3 = 0 \Rightarrow k = -\frac{3}{4}$.

86. Reducing the system to row-echelon form produces

$$x + 5y + z = 0$$
$$y - 2z = 0$$
$$(a - 10)y + (b - 2)z = c$$

$$x + 5y + z = 0$$
$$y - 2z = 0$$
$$(2a + b - 22)z = c.$$

So, you see that

(a) if $2a + b - 22 \neq 0$, then there is exactly one solution.

(b) if $2a + b - 22 = 0$ and $c = 0$, then there is an infinite number of solutions.

(c) if $2a + b - 22 = 0$ and $c \neq 0$, there is no solution.

88. If $c_1 = c_2 = c_3 = 0$, then the system is consistent because $x = y = 0$ is a solution.

90. Multiplying the first equation by c, and the second by a, produces

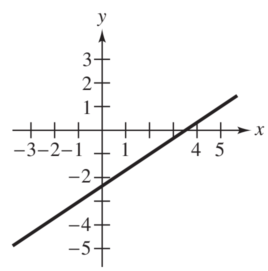
$$acx + bcy = ec$$
$$acx + day = af.$$

Subtracting the second equation from the first yields

$$acx + bcy = ec$$
$$(ad - bc)y = af - ec.$$

So, there is a unique solution if $ad - bc \neq 0$.

92.



The two lines coincide.

$$2x - 3y = 7$$
$$0 = 0$$

Letting $y = t$, $x = \dfrac{7 + 3t}{2}$.

The graph does not change.

94. $21x - 20y = 0$

$13x - 12y = 120$

Subtracting 5 times the second equation from 3 times the first equation produces a new first equation, $-2x = -600$, or $x = 300$. So, $21(300) - 20y = 0$ or $y = 315$, and the solution is: $x = 300$, $y = 315$. The graphs are misleading because they appear to be parallel, but they actually intersect at $(300, 315)$.

Section 1.2 Gaussian Elimination and Gauss-Jordan Elimination

2. Because the matrix has 4 rows and 1 column, it has size 4×1.

4. Because the matrix has 1 row and 1 column, it has size 1×1.

6. Because the matrix has 1 row and 5 columns, it has size 1×5.

8. $\begin{bmatrix} 3 & -1 & -4 \\ -4 & 3 & 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -1 & -4 \\ 5 & 0 & -5 \end{bmatrix}$

Add 3 times Row 1 to Row 2.

10. $\begin{bmatrix} -1 & -2 & 3 & -2 \\ 2 & -5 & 1 & -7 \\ 5 & 4 & -7 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & -2 & 3 & -2 \\ 0 & -9 & 7 & -11 \\ 0 & -6 & 8 & -4 \end{bmatrix}$

Add 2 times Row 1 to Row 2. Then add 5 times Row 1 to Row 3.

12. Because the matrix is in reduced row-echelon form, you can convert back to a system of linear equations

$$x_1 = 2$$
$$x_2 = 3.$$

14. Because the matrix is in row-echelon form, you can convert back to a system of linear equations

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ x_3 &= -1. \end{aligned}$$

Using back-substitution, you have $x_3 = -1$. Letting $x_2 = t$ be the free variable, you can describe the solution as $x_1 = 1 - 2t$, $x_2 = t$, and $x_3 = -1$, where t is any real number.

16. Gaussian elimination produces the following.

$$\begin{bmatrix} 3 & -1 & 1 & 5 \\ 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 2 & 1 & 0 \\ 3 & -1 & 1 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 2 & 0 & -2 \\ 3 & -1 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 2 & 0 & -2 \\ 0 & -1 & -2 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & 0 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -4 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Because the matrix is in row-echelon form, convert back to a system of linear equations.

$$\begin{aligned} x_1 \quad + x_3 &= 2 \\ x_2 + 2x_3 &= 1 \\ x_3 &= 1 \end{aligned}$$

By back-substitution, $x_1 = 1$, $x_2 = -1$, and $x_3 = 1$.

18. Because the fourth row of this matrix corresponds to the equation $0 = 2$, there is no solution to the linear system.

20. Because the leading 1 in the first row is not farther to the left than the leading 1 in the second row, the matrix is *not* in row-echelon form.

22. The matrix satisfies all three conditions in the definition of row-echelon form. However, because the third column does not have zeros above the leading 1 in the third row, the matrix is *not* in reduced row-echelon form.

24. The matrix satisfies all three conditions in the definition of row-echelon form. Moreover, because each column that has a leading 1 (columns one and four) has zeros elsewhere, the matrix is in reduced row-echelon form.

26. The augmented matrix for this system is

$$\begin{bmatrix} 2 & 6 & 16 \\ -2 & -6 & -16 \end{bmatrix}.$$

Use Gauss-Jordan elimination as follows.

$$\begin{bmatrix} 2 & 6 & 16 \\ -2 & -6 & -16 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 8 \\ -2 & -6 & -16 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

Converting back to a system of linear equations, you have

$$x + 3y = 8.$$

Choosing $y = t$ as the free variable, you can describe the solution as $x = 8 - 3t$ and $y = t$, where t is any real number.

28. The augmented matrix for this system is

$$\begin{bmatrix} 2 & -1 & -0.1 \\ 3 & 2 & 1.6 \end{bmatrix}.$$

Gaussian elimination produces the following.

$$\begin{bmatrix} 2 & -1 & -0.1 \\ 3 & 2 & 1.6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{20} \\ 3 & 2 & \frac{8}{5} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{20} \\ 0 & \frac{7}{2} & \frac{7}{4} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{20} \\ 0 & 1 & \frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{5} \\ 0 & 1 & \frac{1}{2} \end{bmatrix}$$

Converting back to a system of equations, the solution is: $x = \frac{1}{5}$ and $y = \frac{1}{2}$.

30. The augmented matrix for this system is

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 6 \\ 3 & -2 & 8 \end{bmatrix}.$$

Gaussian elimination produces the following.

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 6 \\ 3 & -2 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 6 \\ 0 & -8 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -6 \\ 0 & -8 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -6 \\ 0 & 0 & -40 \end{bmatrix}$$

Because the third row corresponds to the equation $0 = -40$, you can conclude that the system has no solution.

32. The augmented matrix for this system is

$$\begin{bmatrix} 3 & -2 & 3 & 22 \\ 0 & 3 & -1 & 24 \\ 6 & -7 & 0 & -22 \end{bmatrix}.$$

Gaussian elimination produces the following.

$$\begin{bmatrix} 3 & -2 & 3 & 22 \\ 0 & 3 & -1 & 24 \\ 6 & -7 & 0 & -22 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{2}{3} & 1 & \frac{22}{3} \\ 0 & 1 & -\frac{1}{3} & 8 \\ 6 & -7 & 0 & -22 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{2}{3} & 1 & \frac{22}{3} \\ 0 & 1 & -\frac{1}{3} & 8 \\ 0 & -3 & -6 & -66 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{2}{3} & 1 & \frac{22}{3} \\ 0 & 1 & -\frac{1}{3} & 8 \\ 0 & 0 & -7 & -42 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{2}{3} & 1 & \frac{22}{3} \\ 0 & 1 & -\frac{1}{3} & 8 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

Back-substitution now yields

$$x_3 = 6$$
$$x_2 = 8 + \tfrac{1}{3}x_3 = 8 + \tfrac{1}{3}(6) = 10$$
$$x_1 = \tfrac{22}{3} + \tfrac{2}{3}x_2 - x_3 = \tfrac{22}{3} + \tfrac{2}{3}(10) - (6) = 8.$$

So, the solution is: $x_1 = 8, x_2 = 10$, and $x_3 = 6$.

34. The augmented matrix for this system is

$$\begin{bmatrix} 1 & 1 & -5 & 3 \\ 1 & 0 & -2 & 1 \\ 2 & -1 & -1 & 0 \end{bmatrix}.$$

Subtracting the first row from the second row yields a new second row.

$$\begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & -1 & 3 & -2 \\ 2 & -1 & -1 & 0 \end{bmatrix}$$

Adding -2 times the first row to the third row yields a new third row.

$$\begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & -1 & 3 & -2 \\ 0 & -3 & 9 & -6 \end{bmatrix}$$

Multiplying the second row by -1 yields a new second row.

$$\begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & 1 & -3 & 2 \\ 0 & -3 & 9 & -6 \end{bmatrix}$$

Adding 3 times the second row to the third row yields a new third row.

$$\begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Adding -1 times the second row to the first row yields a new first row.

$$\begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Converting back to a system of linear equations produces

$$\begin{aligned} x_1 \quad - 2x_3 &= 1 \\ x_2 - 3x_3 &= 2. \end{aligned}$$

Finally, choosing $x_3 = t$ as the free variable, you can describe the solution as $x_1 = 1 + 2t$, $x_2 = 2 + 3t$, and $x_3 = t$, where t is any real number.

36. The augmented matrix for this system is

$$\begin{bmatrix} 1 & 2 & 1 & 8 \\ -3 & -6 & -3 & -21 \end{bmatrix}.$$

Gaussian elimination produces the following matrix.

$$\begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Because the second row corresponds to the equation $0 = 3$, there is no solution to the original system.

38. The augmented matrix for this system is

$$\begin{bmatrix} 2 & 1 & -1 & 2 & -6 \\ 3 & 4 & 0 & 1 & 1 \\ 1 & 5 & 2 & 6 & -3 \\ 5 & 2 & -1 & -1 & 3 \end{bmatrix}.$$

Gaussian elimination produces the following.

$$\begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 3 & 4 & 0 & 1 & 1 \\ 2 & 1 & -1 & 2 & -6 \\ 5 & 2 & -1 & -1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & -11 & -6 & -17 & 10 \\ 0 & -9 & -5 & -10 & 0 \\ 0 & -23 & -11 & -31 & 18 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & 1 & \frac{6}{11} & \frac{17}{11} & -\frac{10}{11} \\ 0 & -9 & -5 & -10 & 0 \\ 0 & -23 & -11 & -31 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & 1 & \frac{6}{11} & \frac{17}{11} & -\frac{10}{11} \\ 0 & 0 & -\frac{1}{11} & \frac{43}{11} & -\frac{90}{11} \\ 0 & 0 & \frac{17}{11} & \frac{50}{11} & -\frac{32}{11} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & 1 & \frac{6}{11} & \frac{17}{11} & -\frac{10}{11} \\ 0 & 0 & 1 & -43 & 90 \\ 0 & 0 & \frac{17}{11} & \frac{50}{11} & -\frac{32}{4} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & 1 & \frac{6}{11} & \frac{17}{11} & -\frac{10}{11} \\ 0 & 0 & 1 & -43 & 90 \\ 0 & 0 & 0 & \frac{781}{11} & -\frac{1562}{11} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & 1 & \frac{6}{11} & \frac{17}{11} & -\frac{10}{11} \\ 0 & 0 & 1 & -43 & 90 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

Back-substitution now yields

$$w = -2$$
$$z = 90 + 43w = 90 + 43(-2) = 4$$
$$y = -\tfrac{10}{11} - \tfrac{6}{11}(z) - \tfrac{17}{11}(w) = -\tfrac{10}{11} - \tfrac{6}{11}(4) - \tfrac{17}{11}(-2) = 0$$
$$x = -3 - 5y - 2z - 6w = -3 - 5(0) - 2(4) - 6(-2) = 1.$$

So, the solution is: $x = 1$, $y = 0$, $z = 4$, and $w = -2$.

40. Using a software program or graphing utility, the augmented matrix reduces to

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

So, the solution is:
$x_1 = 2, x_2 = -1, x_3 = 3, x_4 = 4,$ and $x_5 = 1$.

42. Using a computer software program or graphing utility, you obtain

$$x_1 = 1$$
$$x_2 = -1$$
$$x_3 = 2$$
$$x_4 = 0$$
$$x_5 = -2$$
$$x_6 = 1.$$

44. The corresponding equations are

$$\begin{aligned} x_1 \qquad\quad &= 0 \\ x_2 + x_3 &= 0. \end{aligned}$$

Choosing $x_4 = t$ and $x_3 = t$ as the free variables, you can describe the solution as $x_1 = 0$, $x_2 = -s$, $x_3 = s$, and $x_4 = t$, where s and t are any real numbers.

46. The corresponding equations are all $0 = 0$. So, there are three free variables. So, $x_1 = t$, $x_2 = s$, and $x_3 = r$, where t, s, and r are any real numbers.

48. x = number of \$1 bills

y = number of \$5 bills

z = number of \$10 bills

w = number of \$20 bills

$$\begin{aligned} x + 5y + 10z + 20w &= 95 \\ x + y + z + w &= 26 \\ y - 4z &= 0 \\ x - 2y &= -1 \end{aligned}$$

$$\begin{bmatrix} 1 & 5 & 10 & 20 & 95 \\ 1 & 1 & 1 & 1 & 26 \\ 0 & 1 & -4 & 0 & 0 \\ 1 & -2 & 0 & 0 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 15 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$x = 15$

$y = 8$

$z = 2$

$w = 1$

The server has 15 \$1 bills, 8 \$5 bills, 2 \$10 bills, and one \$20 bill.

50. (a) If A is the *augmented* matrix of a system of linear equations, then the number of equations in this system is three (because it is equal to the number of rows of the augmented matrix). The number of variables is two because it is equal to the number of columns of the augmented matrix minus one.

(b) Using Gaussian elimination on the augmented matrix of a system, you have the following.

$$\begin{bmatrix} 2 & -1 & 3 \\ -4 & 2 & k \\ 4 & -2 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ 0 & 0 & k+6 \\ 0 & 0 & 0 \end{bmatrix}$$

This system is consistent if and only if $k + 6 = 0$, so $k = -6$.

If A is the *coefficient* matrix of a system of linear equations, then the number of equations is three, because it is equal to the number of rows of the coefficient matrix. The number of variables is also three, because it is equal to the number of columns of the coefficient matrix.

Using Gaussian elimination on A you obtain the following coefficient matrix of an equivalent system.

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & k+6 \\ 0 & 0 & 0 \end{bmatrix}$$

Because the homogeneous system is always consistent, the homogeneous system with the coefficient matrix A is consistent for any value of k.

52. Using Gaussian elimination on the augmented matrix, you have the following.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ a & b & c & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & (b-a) & c & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & (a-b+c) & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

From this row reduced matrix you see that the original system has a unique solution.

54. Because the system composed of Equations 1 and 2 is consistent, but has a free variable, this system must have an infinite number of solutions.

56. Use Gauss-Jordan elimination as follows.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

58. Begin by finding all possible first rows

$[0\ \ 0\ \ 0], [0\ \ 0\ \ 1], [0\ \ 1\ \ 0], [0\ \ 1\ \ a], [1\ \ 0\ \ 0], [1\ \ 0\ \ a], [1\ \ a\ \ b], [1\ \ a\ \ 0],$

where a and b are nonzero real numbers. For each of these examine the possible remaining rows.

$$\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}, \begin{bmatrix}0&1&a\\0&0&0\\0&0&0\end{bmatrix},$$

$$\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}, \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}, \begin{bmatrix}1&0&0\\0&0&1\\0&0&0\end{bmatrix}, \begin{bmatrix}1&0&0\\0&1&a\\0&0&0\end{bmatrix},$$

$$\begin{bmatrix}1&a&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}1&a&0\\0&0&1\\0&0&0\end{bmatrix}, \begin{bmatrix}1&a&b\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}1&0&a\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}1&0&a\\0&1&0\\0&0&0\end{bmatrix}$$

60. (a) False. A 4×7 matrix has 4 rows and 7 columns.

(b) True. Reduced row-echelon form of a given matrix is unique while row-echelon form is not. (See also exercise 64 of this section.)

(c) True. See Theorem 1.1 on page 21.

(d) False. Multiplying a row by a *nonzero* constant is one of the elementary row operations. However, multiplying a row of a matrix by a constant $c = 0$ is *not* an elementary row operation. (This would change the system by eliminating the equation corresponding to this row.)

62. No, the row-echelon form is not unique. For instance, $\begin{bmatrix}1&2\\0&1\end{bmatrix}$ and $\begin{bmatrix}1&0\\0&1\end{bmatrix}$. The reduced row-echelon form is unique.

64. First, you need $a \neq 0$ or $c \neq 0$. If $a \neq 0$, then you have

$$\begin{bmatrix}a&b\\c&d\end{bmatrix} \Rightarrow \begin{bmatrix}a&b\\0&-\dfrac{cb}{a}+b\end{bmatrix} \Rightarrow \begin{bmatrix}a&b\\0&ad-bc\end{bmatrix}.$$

So, $ad - bc = 0$ and $b = 0$, which implies that $d = 0$. If $c \neq 0$, then you interchange rows and proceed.

$$\begin{bmatrix}a&b\\c&d\end{bmatrix} \Rightarrow \begin{bmatrix}c&d\\0&-\dfrac{ad}{c}+b\end{bmatrix} \Rightarrow \begin{bmatrix}c&d\\0&ad-bc\end{bmatrix}$$

Again, $ad - bc = 0$ and $d = 0$, which implies that $b = 0$. In conclusion, $\begin{bmatrix}a&b\\c&d\end{bmatrix}$ is row-equivalent to $\begin{bmatrix}1&0\\0&0\end{bmatrix}$ if and only if $b = d = 0$, and $a \neq 0$ or $c \neq 0$.

66. Row reduce the augmented matrix for this system.

$$\begin{bmatrix}2\lambda+9&-5&0\\1&-\lambda&0\end{bmatrix} \Rightarrow \begin{bmatrix}1&-\lambda&0\\2\lambda+9&-5&0\end{bmatrix} \Rightarrow \begin{bmatrix}1&-\lambda&0\\0&2\lambda^2+9\lambda-5&0\end{bmatrix}$$

To have a nontrivial solution you must have the following.

$$2\lambda^2 + 9\lambda - 5 = 0$$

$$(\lambda + 5)(2\lambda - 1) = 0$$

So, if $\lambda = -5$ or $\lambda = \frac{1}{2}$, the system will have nontrivial solutions.

68. A matrix is in reduced row-echelon form if every column that has a leading 1 has zeros in every position above and below its leading 1. A matrix in row-echelon form may have any real numbers above the leading 1's.

70. (a) When a system of linear equations is inconsistent, the row-echelon form of the corresponding augmented matrix will have a row that is all zeros except for the last entry.

(b) When a system of linear equations has infinitely many solutions, the row-echelon form of the corresponding augmented matrix will have a row that consists entirely of zeros or more than one column with no leading 1's. The last column will not contain a leading 1.

Section 1.3 Applications of Systems of Linear Equations

2. (a) Because there are three points, choose a second-degree polynomial, $p(x) = a_0 + a_1x + a_2x^2$.

Then substitute $x = 0, 2,$ and 4 into $p(x)$ and equate the results to $y = 0, -2,$ and 0, respectively.

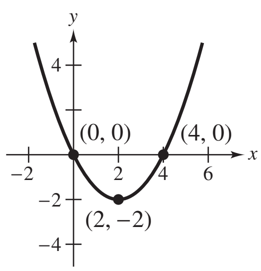
$$\begin{aligned} a_0 + a_1(0) + a_2(0)^2 &= a_0 &&= 0 \\ a_0 + a_1(2) + a_2(2)^2 &= a_0 + 2a_1 + 4a_2 &&= -2 \\ a_0 + a_1(4) + a_2(4)^2 &= a_0 + 4a_1 + 16a_2 &&= 0 \end{aligned}$$

Use Gauss-Jordan elimination on the augmented matrix for this system.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 4 & -2 \\ 1 & 4 & 16 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

So, $p(x) = -2x + \frac{1}{2}x^2$.

(b)



4. (a) Because there are three points, choose a second-degree polynomial, $p(x) = a_0 + a_1x + a_2x^2$.

Then substitute $x = 2, 3,$ and 4 into $p(x)$ and equate the results to $y = 4, 4,$ and 4, respectively.

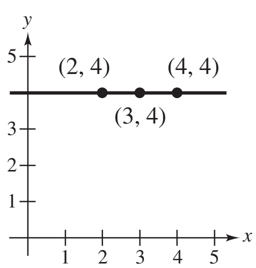
$$\begin{aligned} a_0 + a_1(2) + a_2(2)^2 &= a_0 + 2a_1 + 4a_2 = 4 \\ a_0 + a_1(3) + a_2(3)^2 &= a_0 + 3a_1 + 9a_2 = 4 \\ a_0 + a_1(4) + a_2(4)^2 &= a_0 + 4a_1 + 16a_2 = 4 \end{aligned}$$

Use Gauss-Jordan elimination on the augmented matrix for this system.

$$\begin{bmatrix} 1 & 2 & 4 & 4 \\ 1 & 3 & 9 & 4 \\ 1 & 4 & 16 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

So, $p(x) = 4$.

(b)



6. (a) Because there are four points, choose a third-degree polynomial, $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$. Then substitute $x = 0, 1, 2,$ and 3 into $p(x)$ and equate the results to $y = 42, 0, -40,$ and -72, respectively.

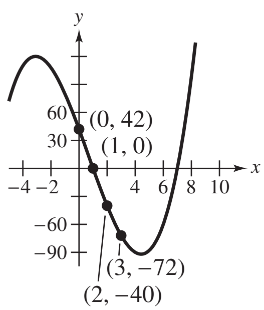
$$\begin{aligned}
a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3 &= a_0 &&= 42\\
a_0 + a_1(1) + a_2(1)^2 + a_3(1)^3 &= a_0 + a_1 + a_2 + a_3 &&= 0\\
a_0 + a_1(2) + a_2(2)^2 + a_3(2)^3 &= a_0 + 2a_1 + 4a_2 + 8a_3 &&= -40\\
a_0 + a_1(3) + a_2(3)^2 + a_3(3)^2 &= a_0 + 3a_1 + 9a_2 + 27a_3 &&= -72
\end{aligned}$$

Use Gauss-Jordan elimination on the augmented matrix for this system.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 42 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 4 & 8 & -40 \\ 1 & 3 & 9 & 27 & -72 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 42 \\ 0 & 1 & 0 & 0 & -41 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

So, $p(x) = 42 - 41x - 2x^2 + x^3$.

(b)



8. (a) Because there are five points, choose a fourth-degree polynomial, $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$. Then substitute $x = -4, 0, 4, 6,$ and 8 into $p(x)$ and equate the results to $y = 18, 1, 0, 28,$ and 135, respectively.

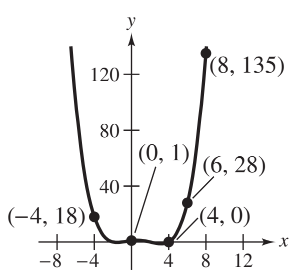
$$\begin{aligned}
a_0 + a_1(-4) + a_2(-4)^2 + a_3(-4)^3 + a_4(-4)^4 &= a_0 - 4a_1 + 16a_2 - 64a_3 + 256a_4 &&= 18\\
a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3 + a_4(0)^4 &= a_0 &&= 1\\
a_0 + a_1(4) + a_2(4)^2 + a_3(4)^3 + a_4(4)^4 &= a_0 + 4a_1 + 16a_2 + 64a_3 + 256a_4 &&= 0\\
a_0 + a_1(6) + a_2(6)^2 + a_3(6)^3 + a_4(6)^4 &= a_0 + 6a_1 + 36a_2 + 216a_3 + 1296a_4 &&= 28\\
a_0 + a_1(8) + a_2(8)^2 + a_3(8)^3 + a_4(8)^4 &= a_0 + 8a_1 + 64a_2 + 512a_3 + 4096a_4 &&= 135
\end{aligned}$$

Use Gauss-Jordan elimination on the augmented matrix for this system.

$$\begin{bmatrix} 1 & -4 & 16 & -64 & 256 & 18 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 4 & 16 & 64 & 256 & 0 \\ 1 & 6 & 36 & 216 & 1296 & 28 \\ 1 & 8 & 64 & 512 & 4096 & 135 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 1 & 0 & -\frac{3}{16} \\ 0 & 0 & 0 & 0 & 1 & \frac{1}{16} \end{bmatrix}$$

So, $p(x) = 1 + \frac{3}{4}x - \frac{1}{2}x^2 - \frac{3}{16}x^3 + \frac{1}{16}x^4 = \frac{1}{16}\left(16 + 12x - 8x^2 - 3x^3 + x^4\right)$.

(b)



10. (a) Let $z = x - 2012$. Because there are four points, choose a third-degree polynomial, $p(z) = a_0 + a_1z + a_2z^2 + a_3z^3$. Then substitute $z = 0, 1, 2,$ and 3 into $p(z)$ and equate the results to $y = 150, 180, 240,$ and 360 respectively.

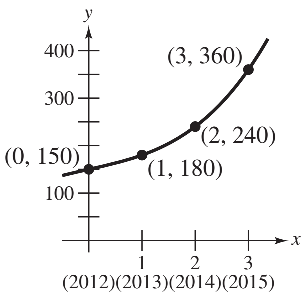
$$\begin{aligned}
a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3 &= a_0 &&= 150\\
a_0 + a_1(1) + a_2(1)^2 + a_3(1)^3 &= a_0 + a_1 + a_2 + a_3 &&= 180\\
a_0 + a_1(2) + a_2(2)^2 + a_3(2)^3 &= a_0 + 2a_1 + 4a_2 + 8a_3 &&= 240\\
a_0 + a_1(3) + a_2(3)^2 + a_3(3)^3 &= a_0 + 3a_1 + 9a_2 + 27a_3 &&= 360
\end{aligned}$$

Use Gauss-Jordan elimination on the augmented matrix for this system.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 150\\ 1 & 1 & 1 & 1 & 180\\ 1 & 2 & 4 & 8 & 240\\ 1 & 3 & 9 & 27 & 360 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 150\\ 0 & 1 & 0 & 0 & 25\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

So, $p(z) = 150 + 25z + 5z^3$, or $p(x) = 150 + 25(x - 2012) + 5(x - 2012)^3$.

(b)



12. (a) Because there are four points, choose a third-degree polynomial, $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$. Then substitute $x = 1, 1.189, 1.316,$ and 1.414 into $p(x)$ and equate the results to $y = 1, 1.587, 2.080,$ and 2.520, respectively.

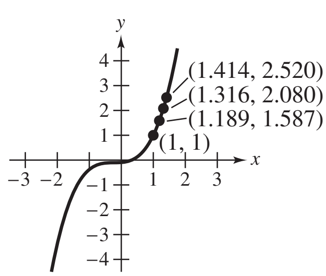
$$\begin{aligned}
a_0 + a_1(1) + a_2(1)^2 + a_3(1)^3 &= a_0 + a_1 + a_2 + a_3 &&= 1\\
a_0 + a_1(1.189) + a_2(1.189)^2 + a_3(1.189)^3 &\approx a_0 + 1.189a_1 + 1.414a_2 + 1.681a_3 &&= 1.587\\
a_0 + a_1(1.316) + a_2(1.316)^2 + a_3(1.316)^3 &\approx a_0 + 1.316a_1 + 1.732a_2 + 2.279a_3 &&= 2.080\\
a_0 + a_1(1.414) + a_2(1.414)^2 + a_3(1.414)^3 &\approx a_0 + 1.414a_1 + 1.999a_2 + 2.827a_3 &&= 2.520
\end{aligned}$$

Use Gauss-Jordan elimination on the augmented matrix for this system.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1\\ 1 & 1.189 & 1.414 & 1.681 & 1.587\\ 1 & 1.316 & 1.732 & 2.279 & 2.080\\ 1 & 1.414 & 1.999 & 2.827 & 2.520 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -0.095\\ 0 & 1 & 0 & 0 & 0.103\\ 0 & 0 & 1 & 0 & 0.405\\ 0 & 0 & 0 & 1 & 0.587 \end{bmatrix}$$

So, $p(x) \approx -0.095 + 0.103x + 0.405x^2 + 0.587x^3$.

(b)



14. Choosing a second-degree polynomial approximation $p(x) = a_0 + a_1x + a_2x^2$, substitute $x = 1, 2,$ and 4 into $p(x)$ and equate the results to $y = 0, 1,$ and 2, respectively.

$$\begin{aligned} a_0 + a_1 + a_2 &= 0 \\ a_0 + 2a_1 + 4a_2 &= 1 \\ a_0 + 4a_1 + 16a_2 &= 2 \end{aligned}$$

The solution to this system is $a_0 = -\frac{4}{3}$, $a_1 = \frac{3}{2}$, and $a_2 = -\frac{1}{6}$.

So, $p(x) = -\frac{4}{3} + \frac{3}{2}x - \frac{1}{6}x^2$.

Finally, to estimate $\log_2 3$, calculate $p(3) = -\frac{4}{3} + \frac{3}{2}(3) - \frac{1}{6}(3)^2 = \frac{5}{3}$.

16. Assume that the equation of the circle is $x^2 + ax + y^2 + by - c = 0$. Because each of the given points lie on the circle, you have the following linear equations.

$$\begin{aligned} (-5)^2 + a(-5) + (1)^2 + b(1)^2 - c &= -5a + b - c + 26 = 0 \\ (-3)^2 + a(-3) + (2)^2 + b(2) - c &= -3a + 2b - c + 13 = 0 \\ (-1)^2 + a(-1) + (1)^2 + b(1) - c &= -a + b - c + 2 = 0 \end{aligned}$$

Use Gauss-Jordan elimination on the system.

$$\begin{bmatrix} -5 & 1 & -1 & -26 \\ -3 & 2 & -1 & -13 \\ -1 & 1 & -1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

So, the equation of the circle is $x^2 - 6x + y^2 + y + 3 = 0$, or $(x - 3)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{25}{4}$.

18. (a) Letting $z = \dfrac{x - 1970}{10}$, the four data points are $(0, 205), (1, 227), (2, 249),$ and $(3, 282)$. Let $p(z) = a_0 + a_1z + a_2z^2 + a_3z^3$. Substituting the points into $p(z)$ produces the following system of linear equations.

$$\begin{aligned} a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3 &= a_0 = 205 \\ a_0 + a_1(1) + a_2(1)^2 + a_3(1)^3 &= a_0 + a_1 + a_2 + a_3 = 227 \\ a_0 + a_1(2) + a_2(2)^2 + a_3(2)^3 &= a_0 + 2a_1 + 4a_2 + 8a_3 = 249 \\ a_0 + a_1(3) + a_2(3)^2 + a_3(3)^3 &= a_0 + 3a_1 + 9a_2 + 27a_3 = 282 \end{aligned}$$

Form the augmented matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 205 \\ 1 & 1 & 1 & 1 & 227 \\ 1 & 2 & 4 & 8 & 249 \\ 1 & 3 & 9 & 27 & 282 \end{bmatrix}$$

and use Gauss-Jordan elimination to obtain the equivalent reduced row-echelon matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 205 \\ 0 & 1 & 0 & 0 & \frac{77}{3} \\ 0 & 0 & 1 & 0 & -\frac{11}{2} \\ 0 & 0 & 0 & 1 & \frac{11}{6} \end{bmatrix}$$

So, the cubic polynomial is $p(z) = 205 + \dfrac{77}{3}z - \dfrac{11}{2}z^2 + \dfrac{11}{6}z^3$.

Because $z = \dfrac{x - 1970}{10}$, $p(x) = 205 + \dfrac{77}{3}\left(\dfrac{x - 1970}{10}\right) - \dfrac{11}{2}\left(\dfrac{x - 1970}{10}\right) + \dfrac{11}{6}\left(\dfrac{x - 1970}{10}\right)^3$.

(b) To estimate the population in 2010, let $x = 2010$. $p(2010) = 205 + \dfrac{77}{3}(4) - \dfrac{11}{2}(4)^2 + \dfrac{11}{6}(4)^3 = 337$ million, which is greater than the actual population of 309 million.

20. (a) Letting $z = x - 2000$, the five points $(6, 348.7), (7, 378.8), (8, 405.6), (9, 408.2)$, and $(10, 421.8)$.

Let $p(z) = a_0 + a_1z + a_2z^2 + a_3z^3 + a_4z^4$.

$$\begin{aligned}
a_0 + a_1(6) + a_2(6)^2 + a_3(6)^3 + a_4(6)^4 &= a_0 + 6a_1 + 36a_2 + 216a_3 + 1296a_4 = 348.7\\
a_0 + a_1(7) + a_2(7)^2 + a_3(7)^3 + a_4(7)^4 &= a_0 + 7a_1 + 49a_2 + 343a_3 + 2401a_4 = 378.8\\
a_0 + a_1(8) + a_2(8)^2 + a_3(8)^3 + a_4(8)^4 &= a_0 + 8a_1 + 64a_2 + 512a_3 + 4096a_4 = 405.6\\
a_0 + a_1(9) + a_2(9)^2 + a_3(9)^3 + a_4(9)^4 &= a_0 + 9a_1 + 81a_2 + 729a_3 + 6561a_4 = 408.2\\
a_0 + a_1(10) + a_2(10)^2 + a_3(10)^3 + a_4(10)^4 &= a_0 + 10a_1 + 100a_2 + 1000a_3 + 10{,}000a_4 = 421.8
\end{aligned}$$

(b) Use Gauss-Jordan elimination to solve the system.

$$\begin{bmatrix} 1 & 6 & 36 & 216 & 1296 & 348.7\\ 1 & 7 & 49 & 343 & 2401 & 378.8\\ 1 & 8 & 64 & 512 & 4096 & 405.6\\ 1 & 9 & 81 & 729 & 6561 & 408.2\\ 1 & 10 & 100 & 1000 & 10{,}000 & 421.8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 8337.8\\ 0 & 1 & 0 & 0 & 0 & -4313.89\\ 0 & 0 & 1 & 0 & 0 & 854.563\\ 0 & 0 & 0 & 1 & 0 & -73.608\\ 0 & 0 & 0 & 0 & 1 & 2.338 \end{bmatrix}$$

So, $p(z) = 8337.8 - 4313.89z + 854.563z^2 - 73.608z^3 + 2.338z^4$. Because $z = x - 2000$,

$$p(x) = 8337.8 - 4313.89(x - 2000) + 854.563(x - 2000)^2 - 73.608(x - 2000)^3 + 2.338(x - 2000)^4.$$

To determine the reasonableness of the model for years after 2010, compare the predicted values for 2011–2013 to the actual values.

x	2011	2012	2013
$p(x)$	537.8	903.4	1722.3
Actual	447.0	469.2	476.2

The model does not produce reasonable outcomes after 2010.

22. (a) Each of the network's four junctions gives rise to a linear equation as shown below.

$$\begin{aligned}
\textbf{input} &= \textbf{output}\\
300 &= x_1 + x_2\\
x_1 + x_3 &= x_4 + 150\\
x_2 + 200 &= x_3 + x_5\\
x_4 + x_5 &= 350
\end{aligned}$$

Rearrange these equations, form the augmented matrix, and use Gauss-Jordan elimination.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 300\\ 1 & 0 & 1 & -1 & 0 & 150\\ 0 & 1 & -1 & 0 & -1 & -200\\ 0 & 0 & 0 & 1 & 1 & 350 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 500\\ 0 & 1 & -1 & 0 & -1 & -200\\ 0 & 0 & 0 & 1 & 1 & 350\\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Letting $x_5 = t$ and $x_3 = s$ be the free variables, you have

$x_1 = 500 - s - t$

$x_2 = -200 + s + t$

$x_3 = s$

$x_4 = 350 - t$

$x_5 = t$, where t and s are any real numbers.

(b) If $x_2 = 200$ and $x_3 = 50$, then you have $s = 50$ and $t = 350$.

So, the solution is: $x_1 = 100$, $x_2 = 200$, $x_3 = 50$, $x_4 = 0$, and $x_5 = 350$.

(c) If $x_2 = 150$ and $x_3 = 0$, then you have $s = 0$ and $t = 350$.

So, the solution is: $x_1 = 150$, $x_2 = 150$, $x_3 = 0$, $x_4 = 0$, and $x_5 = 350$.

24. (a) Each of the network's six junctions gives rise to a linear equation as shown below.

$$\begin{aligned} \textbf{input} &= \textbf{output} \\ 600 &= x_1 + x_3 \\ x_1 &= x_2 + x_4 \\ x_2 + x_5 &= 500 \\ x_3 + x_6 &= 600 \\ x_4 + x_7 &= x_6 \\ 500 &= x_5 + x_7 \end{aligned}$$

Rearrange these equations, form the augmented matrix, and use Gauss-Jordan elimination.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 600 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 500 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 600 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 500 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 600 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 500 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Letting $x_7 = t$ and $x_6 = s$ be the free variables, you have

$$\begin{aligned} x_1 &= s \\ x_2 &= t \\ x_3 &= 600 - s \\ x_4 &= s - t \\ x_5 &= 500 - t \\ x_6 &= s \\ x_7 &= t, \end{aligned}$$

where s and t are any real numbers.

(b) If $x_1 = x_2 = 100$, then the solution is $x_1 = 100$, $x_2 = 100$, $x_3 = 500$, $x_4 = 0$, $x_5 = 400$, $x_6 = 100$, and $x_7 = 100$.

(c) If $x_6 = x_7 = 0$, then the solution is $x_1 = 0$, $x_2 = 0$, $x_3 = 600$, $x_4 = 0$, $x_5 = 500$, $x_6 = 0$, and $x_7 = 0$.

(d) If $x_5 = 1000$ and $x_6 = 0$, then the solution is $x_1 = 0$, $x_2 = -500$, $x_3 = 600$, $x_4 = 500$, $x_5 = 1000$, $x_6 = 0$, and $x_7 = -500$.

26. Applying Kirchoff's first law to three of the four junctions produces

$$\begin{aligned} I_1 + I_3 &= I_2 \\ I_1 + I_4 &= I_2 \\ I_3 + I_6 &= I_5 \end{aligned}$$

and applying the second law to the three paths produces

$$\begin{aligned} R_1I_1 + R_2I_2 &= 3I_1 + 2I_2 = 14 \\ R_2I_2 + R_4I_4 + R_5I_5 + R_3I_3 &= 2I_2 + 2I_4 + I_5 + 4I_3 = 25 \\ R_5I_5 + R_6I_6 &= I_5 + I_6 = 8. \end{aligned}$$

Rearrange these equations, form the augmented matrix, and use Gauss-Jordan elimination.

$$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 3 & 2 & 0 & 0 & 0 & 0 & 14 \\ 0 & 2 & 4 & 2 & 1 & 0 & 25 \\ 0 & 0 & 0 & 0 & 1 & 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

So, the solution is: $I_1 = 2$, $I_2 = 4$, $I_3 = 2$, $I_4 = 2$, $I_5 = 5$, and $I_6 = 3$.

28. (a) For a set of n points with distinct x-values, substitute the points into the polynomial $p(x) = a_0 + a_1x + \cdots + a_{n-1}x^{n-1}$. This creates a system of linear equations in $a_0, a_1, \cdots a_{n-1}$. Solving the system gives values for the coefficients a_n, and the resulting polynomial fits the original points.

(b) In a network, the total flow into a junction is equal to the total flow out of a junction. So, each junction determines an equation, and the set of equations for all the junctions in a network forms a linear system. In an electrical network, Kirchhoff's Laws are used to determine additional equations for the system.

30.
$$T_1 = \frac{50 + 25 + T_2 + T_3}{4}$$
$$T_2 = \frac{50 + 25 + T_1 + T_4}{4}$$
$$T_3 = \frac{25 + 0 + T_1 + T_4}{4}$$
$$T_4 = \frac{25 + 0 + T_2 + T_3}{4}$$
$$\Rightarrow \begin{aligned} 4T_1 - T_2 - T_3 \phantom{{}- T_4} &= 75 \\ -T_1 + 4T_2 - T_4 &= 75 \\ -T_1 + 4T_3 - T_4 &= 25 \\ -T_2 - T_3 + 4T_4 &= 25 \end{aligned}$$

Use Gauss-Jordan elimination to solve this system.

$$\begin{bmatrix} 4 & -1 & -1 & 0 & 75 \\ -1 & 4 & 0 & -1 & 75 \\ -1 & 0 & 4 & -1 & 25 \\ 0 & -1 & -1 & 4 & 25 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 31.25 \\ 0 & 1 & 0 & 0 & 31.25 \\ 0 & 0 & 1 & 0 & 18.75 \\ 0 & 0 & 0 & 1 & 18.75 \end{bmatrix}$$

So, $T_1 = 31.25°\text{C}, T_2 = 31.25°\text{C}, T_3 = 18.75°\text{C}$, and $T_4 = 18.75°\text{C}$.

32.
$$\frac{3x^2 - 7x - 12}{(x + 4)(x - 4)^2} = \frac{A}{x + 4} + \frac{B}{x - 4} + \frac{C}{(x - 4)^2}$$

$$3x^2 - 7x - 12 = A(x - 4)^2 + B(x + 4)(x - 4) + C(x + 4)$$
$$3x^2 - 7x - 12 = Ax^2 - 8Ax + 16A + Bx^2 - 16B + Cx + 4C$$
$$3x^2 - 7x - 12 = (A + B)x^2 + (-8A + C)x + 16A - 16B + 4C$$

So,
$$\begin{aligned} A + B \phantom{{}+ 4C} &= 3 \\ -8A + C &= -7 \\ 16A - 16B + 4C &= -12. \end{aligned}$$

Use Gauss-Jordan elimination to solve the system.

$$\begin{bmatrix} 1 & 1 & 0 & 3 \\ -8 & 0 & 1 & -7 \\ 16 & -16 & 4 & -12 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

The solution is: $A = 1,\ B = 2$, and $C = 1$.

So, $\dfrac{3x^2 - 7x - 12}{(x + 4)(x - 4)^2} = \dfrac{1}{x + 4} + \dfrac{2}{x - 4} + \dfrac{1}{(x - 4)^2}$

34. Use Gauss-Jordan elimination to solve the system.

$$\begin{bmatrix} 0 & 2 & 2 & -2 \\ 2 & 0 & 1 & -1 \\ 2 & 1 & 0 & 100 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 25 \\ 0 & 1 & 0 & 50 \\ 0 & 0 & 1 & -51 \end{bmatrix}$$

So, $x = 25,\ y = 50$, and $\lambda = -51$.

36.
$$\begin{aligned} 2y + 2\lambda + 2 &= 0 \\ 2x + \lambda + 1 &= 0 \\ 2x + y - 100 &= 0 \end{aligned}$$

The augmented matrix for this system is

$$\begin{bmatrix} 0 & 2 & 2 & -2 \\ 2 & 0 & 1 & -1 \\ 2 & 1 & 0 & 100 \end{bmatrix}.$$

Gauss-Jordan elimination produces the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 25 \\ 0 & 1 & 0 & 50 \\ 0 & 0 & 1 & -51 \end{bmatrix}.$$

So, $x = 25$, $y = 50$, and $\lambda = -51$.

38. To begin, substitute $x = -1$ and $x = 1$ into $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and equate the results to $y = 2$ and $y = -2$, respectively.

$$\begin{aligned} a_0 - a_1 + a_2 - a_3 &= 2 \\ a_0 + a_1 + a_2 + a_3 &= -2 \end{aligned}$$

Then, differentiate p, yielding $p'(x) = a_1 + 2a_2x + 3a_3x^2$. Substitute $x = -1$ and $x = 1$ into $p'(x)$ and equate the results to 0.

$$\begin{aligned} a_1 - 2a_2 + 3a_3 &= 0 \\ a_1 + 2a_2 + 3a_3 &= 0 \end{aligned}$$

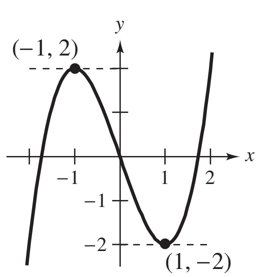
Combining these four equations into one system and forming the augmented matrix, you obtain

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 2 \\ 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & -2 & 3 & 0 \\ 0 & 1 & 2 & 3 & 0 \end{bmatrix}.$$

Use Gauss-Jordan elimination to find the equivalent reduced row-echelon matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

So, $p(x) = -3x + x^3$. The graph of $y = p(x)$ is shown below.



40. Let

$p_1(x) = a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1}$ and $p_2(x) = b_0 + b_1x + b_2x^2 + \cdots + b_{n-1}x^{n-1}$

be two different polynomials that pass through the n given points. The polynomial

$p_1(x) - p_2(x) = (a_0 - b_0) + (a_1 - b_1)x + (a_2 - b_2)x^2 + \cdots + (a_{n-1} - b_{n-1})x^{n-1}$

is zero for these n values of x. So, $a_0 = b_0,\ a_1 = b_1,\ a_2 = b_2,\ \ldots,\ a_{n-1} = b_{n-1}$.

Therefore, there is only one polynomial function of degree $n - 1$ (or less) whose graph passes through n points in the plane with distinct x-coordinates.

42. Choose a fourth-degree polynomial and substitute $x = 1, 2, 3,$ and 4 into $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$.

However, when you substitute $x = 3$ into $p(x)$ and equate it to $y = 2$ and $y = 3$ you get the contradictory equations

$$a_0 + 3a_1 + 9a_2 + 27a_3 + 81a_4 = 2$$
$$a_0 + 3a_1 + 9a_2 + 27a_3 + 81a_4 = 3$$

and must conclude that the system containing these two equations will have no solution. Also, y is not a function of x because the x-value of 3 is repeated. By similar reasoning, you cannot choose $p(y) = b_0 + b_1y + b_2y^2 + b_3y^3 + b_4y^4$ because $y = 1$ corresponds to both $x = 1$ and $x = 2$.

Review Exercises for Chapter 1

2. Because the equation cannot be written in the form $a_1x + a_2y = b$, it is *not* linear in the variables x and y.

4. Because the equation is in the form $a_1x + a_2y = b$, it is linear in the variables x and y.

6. Because the equation is in the form $a_1x + a_2y = b$, it is linear in the variables x and y.

8. Choosing x_2 and x_3 as the free variables and letting $x_2 = s$ and $x_3 = t$, you have

$$\begin{aligned} 3x_1 + 2s - 4t &= 0 \\ 3x_1 &= -2s + 4t \\ x_1 &= \tfrac{1}{3}(-2s + 4t). \end{aligned}$$

10. Row reduce the augmented matrix for this system.

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

Converting back to a linear system, the solution is $x = 2$ and $y = -3$.

12. Rearrange the equations, form the augmented matrix, and row reduce.

$$\begin{bmatrix} 1 & -1 & 3 \\ 4 & -1 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 3 \\ 0 & 3 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -\frac{2}{3} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \frac{7}{3} \\ 0 & 1 & -\frac{2}{3} \end{bmatrix}.$$

Converting back to a linear system, you obtain the solution $x = \frac{7}{3}$ and $y = -\frac{2}{3}$.

14. Rearrange the equations, form the augmented matrix, and row reduce.

$$\begin{bmatrix} -5 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ -5 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & -4 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Converting back to a linear system, the solution is: $x = 0$ and $y = 0$.

16. Row reduce the augmented matrix for this system.

$$\begin{bmatrix} 40 & 30 & 24 \\ 20 & 15 & -14 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & \frac{3}{4} & \frac{3}{5} \\ 20 & 15 & -14 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & \frac{3}{4} & \frac{3}{5} \\ 0 & 0 & -26 \end{bmatrix}$$

Because the second row corresponds to the false statement $0 = -26$, the system has no solution.

18. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} \frac{1}{3} & \frac{4}{7} & 3 \\ 2 & 3 & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 7 \end{bmatrix}$$

So, the solution is: $x = -3,\ y = 7$.

20. Multiplying both equations by 100 and forming the augmented matrix produces

$$\begin{bmatrix} 20 & -10 & 7 \\ 40 & -50 & -1 \end{bmatrix}.$$

Gauss-Jordan elimination yields the following.

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{7}{20} \\ 40 & -50 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & \frac{7}{20} \\ 0 & -30 & -15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & \frac{7}{20} \\ 0 & 1 & \frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \frac{3}{5} \\ 0 & 1 & \frac{1}{2} \end{bmatrix}$$

So, the solution is: $x = \frac{3}{5}$ and $y = \frac{1}{2}$.

22. Because the matrix has 3 rows and 2 columns, it has size 3×2.

24. This matrix corresponds to the system

$-2x_1 + 3x_2 = 0.$

Choosing $x_2 = t$ as a free variable, you can describe the solution as $x_1 = \frac{3}{2}t$ and $x_2 = t$, where t is a real number.

26. This matrix corresponds to the system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ 0 &= 1. \end{aligned}$$

Because the second equation is not possible, the system has no solution.

28. The matrix satisfies all three conditions in the definition of row-echelon form. Because each column that has a leading 1 (columns 1 and 4) has zeros elsewhere, the matrix is in reduced row-echelon form.

30. The matrix satisfies all three conditions in the definition of row-echelon form. Because each column that has a leading 1 (columns 2 and 3) has zeros elsewhere, the matrix is in reduced row-echelon form.

32. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 4 & 2 & 1 & 18 \\ 4 & -2 & -2 & 28 \\ 2 & -3 & 2 & -8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -6 \end{bmatrix}$$

So, the solution is: $x = 5$, $y = 2$, and $z = -6$.

34. Use the Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 2 & 1 & 2 & 4 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 & \frac{3}{2} \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Choosing $z = t$ as the free variable, you can describe the solution as $x = \frac{3}{2} - 2t$, $y = 1 + 2t$, and $z = t$, where t is any real number.

36. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 2 & 0 & 6 & -9 \\ 3 & -2 & 11 & -16 \\ 3 & -1 & 7 & -11 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & -\frac{3}{4} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} \end{bmatrix}$$

So, the solution is: $x = -\frac{3}{4}$, $y = 0$, and $z = -\frac{5}{4}$.

38. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 2 & 5 & -19 & 34 \\ 3 & 8 & -31 & 54 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & -5 & 6 \end{bmatrix}$$

Choosing $x_3 = t$ as the free variable, you can describe the solution as $x_1 = 2 - 3t$, $x_2 = 6 + 5t$, and $x_3 = t$, where t is any real number.

40. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & 5 & 3 & 0 & 0 & 14 \\ 0 & 4 & 2 & 5 & 0 & 3 \\ 0 & 0 & 3 & 8 & 6 & 16 \\ 2 & 4 & 0 & 0 & -2 & 0 \\ 2 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

So, the solution is: $x_1 = 2$, $x_2 = 0$, $x_3 = 4$, $x_4 = -1$, and $x_5 = 2$.

42. Using a graphing utility, the augmented matrix reduces to

$$\begin{bmatrix} 1 & 0 & -0.533 & 0 \\ 0 & 1 & 1.733 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Because $0 \neq 1$, the system has no solution.

44. Using a graphing utility, the augmented matrix reduces to

$$\begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The system is inconsistent, so there is no solution.

46. Using a graphing utility, the augmented matrix reduces to

$$\begin{bmatrix} 1 & 0 & 0 & 1.5 & 0 \\ 0 & 1 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & 0.5 & 0 \end{bmatrix}.$$

Choosing $w = t$ as the free variable, you can describe the solution as $x = -1.5t$, $y = -0.5t$, $z = -0.5t$, $w = t$, where t is any real number.

48. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 2 & 4 & -7 & 0 \\ 1 & -3 & 9 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \frac{3}{2} & 0 \\ 0 & 1 & -\frac{5}{2} & 0 \end{bmatrix}$$

Letting $x_3 = t$ be the free variable, you have $x_1 = -\frac{3}{2}t$, $x_2 = \frac{5}{2}t$, and $x_3 = t$, where t is any real number.

50. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & 3 & 5 & 0 \\ 1 & 4 & \frac{1}{2} & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \frac{37}{2} & 0 \\ 0 & 1 & -\frac{9}{2} & 0 \end{bmatrix}$$

Choosing $x_3 = t$ as the free variable, you can describe the solution as $x_1 = -\frac{37}{2}t$, $x_2 = \frac{9}{2}t$, and $x_3 = t$, where t is any real number.

52. Use Gaussian elimination on the augmented matrix.

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ -1 & 1 & -1 & 0 \\ 1 & k & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & (k+1) & -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & (k+1) & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

So, there will be exactly one solution (the trivial solution $x = y = z = 0$) if and only if $k \neq -1$.

54. Form the augmented matrix for the system.

$$\begin{bmatrix} 2 & -1 & 1 & a \\ 1 & 1 & 2 & b \\ 0 & 3 & 3 & c \end{bmatrix}$$

Use Gaussian elimination to reduce the matrix to row-echelon form.

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{a}{2} \\ 1 & 1 & 2 & b \\ 0 & 3 & 3 & c \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \frac{a}{2} \\ 0 & \frac{3}{2} & \frac{3}{2} & \frac{2b-a}{2} \\ 0 & 3 & 3 & c \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{a}{2} \\ 0 & 1 & 1 & \frac{2b-a}{3} \\ 0 & 3 & 3 & c \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{a}{2} \\ 0 & 1 & 1 & \frac{2b-a}{3} \\ 0 & 0 & 0 & c-2b+a \end{bmatrix}$$

(a) If $c - 2b + a \neq 0$, then the system has no solution.

(b) The system cannot have one solution.

(c) If $c - 2b + a = 0$, then the system has infinitely many solutions

56. Find all possible first rows, where a and b are nonzero real numbers.

$[0\ \ 0\ \ 0]$, $[0\ \ 0\ \ 1]$, $[0\ \ 1\ \ 0]$, $[0\ \ 1\ \ a]$, $[1\ \ 0\ \ 0]$, $[1\ \ a\ \ 0]$, $[1\ \ a\ \ b]$, $[1\ \ 0\ \ a]$

For each of these, examine the possible second rows.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 1 & a \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & a \end{bmatrix},$$

$$\begin{bmatrix} 1 & a & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & a & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & a & b \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & a \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \end{bmatrix}$$

58. Use Gaussian elimination on the augmented matrix.

$$\begin{bmatrix} (\lambda+2) & -2 & 3 & 0 \\ -2 & (\lambda-1) & 6 & 0 \\ 1 & 2 & \lambda & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & \lambda & 0 \\ 0 & \lambda+3 & 6+2\lambda & 0 \\ 0 & -2\lambda-6 & -\lambda^2-2\lambda+3 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & \lambda & 0 \\ 0 & \lambda+3 & 6+2\lambda & 0 \\ 0 & 0 & (\lambda^2-2\lambda-15) & 0 \end{bmatrix}$$

So, you need $\lambda^2 - 2\lambda - 15 = (\lambda - 5)(\lambda + 3) = 0$, which implies $\lambda = 5$ or $\lambda = -3$.

60. (a) True. A homogeneous system of linear equations is always consistent, because there is always a trivial solution, *i.e.*, when all variables are equal to zero. See Theorem 1.1 on page 21.

(b) False. Consider, for example, the following system (with three variables and two equations).

$$\begin{aligned} x + y - z &= 2 \\ -2x - 2y + 2z &= 1. \end{aligned}$$

It is easy to see that this system has *no* solution.

62. From the following chart, you obtain a system of equations.

	A	B	C
Mixture X	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$
Mixture Y	0	0	1
Mixture Z	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Desired Mixture	$\frac{6}{27}$	$\frac{8}{27}$	$\frac{13}{27}$

$$\left.\begin{aligned} \tfrac{1}{5}x + \tfrac{1}{3}z &= \tfrac{6}{27} \\ \tfrac{2}{5}x + \tfrac{1}{3}z &= \tfrac{8}{27} \end{aligned}\right\} x = \tfrac{10}{27},\ z = \tfrac{12}{27}$$

$$\tfrac{2}{5}x + y + \tfrac{1}{3}z = \tfrac{13}{27} \Rightarrow y = \tfrac{5}{27}$$

To obtain the desired mixture, use 10 gallons of spray X, 5 gallons of spray Y, and 12 gallons of spray Z.

64. $$\frac{3x^2 + 3x - 2}{(x+1)^2(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x+1)^2}$$

$$\begin{aligned} 3x^2 + 3x - 2 &= A(x+1)(x-1) + B(x+1)^2 + C(x-1) \\ 3x^2 + 3x - 2 &= Ax^2 - A + Bx^2 + 2Bx + B + Cx - C \\ 3x^2 + 3x - 2 &= (A+B)x^2 + (2B+C)x - A + B - C \end{aligned}$$

So,
$$\begin{aligned} A + B \phantom{{}+C} &= 3 \\ 2B + C &= 3 \\ -A + B - C &= -2. \end{aligned}$$

Use Gauss-Jordan elimination to solve the system.

$$\begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 2 & 1 & 3 \\ -1 & 1 & -1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

The solution is: $A = 2$, $B = 1$, and $C = 1$.

So, $$\frac{3x^2 + 3x - 2}{(x+1)^2(x-1)} = \frac{2}{x+1} + \frac{1}{x-1} + \frac{1}{(x+1)^2}.$$

66. (a) Because there are four points, choose a third-degree polynomial, $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$.

By substituting the values at each point into this equation, you obtain the system

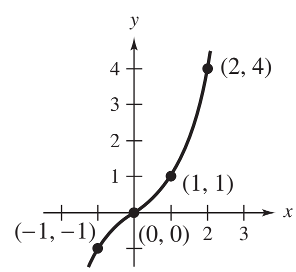
$$\begin{aligned} a_0 - a_1 + a_2 - a_3 &= -1 \\ a_0 &= 0 \\ a_0 + a_1 + a_2 + a_3 &= 1 \\ a_0 + 2a_1 + 4a_2 + 8a_3 &= 4. \end{aligned}$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & -1 & 1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{bmatrix}$$

So, $p(x) = \frac{2}{3}x + \frac{1}{3}x^3$.

(b)



68. Substituting the points, (1, 0), (2, 0), (3, 0), and (4, 0) into the polynomial $p(x)$ yields the system

$$\begin{aligned} a_0 + a_1 + a_2 + a_3 &= 0 \\ a_0 + 2a_1 + 4a_2 + 8a_3 &= 0 \\ a_0 + 3a_1 + 9a_2 + 27a_3 &= 0 \\ a_0 + 4a_1 + 16a_2 + 64a_3 &= 0. \end{aligned}$$

Gaussian elimination shows that the only solution is $a_0 = a_1 = a_2 = a_3 = 0$.

70. (a) When $t = 0, s = 160$: $\frac{1}{2}a(0)^2 + v_0(0) + s_0 = 160 \Rightarrow \quad s_0 = 160$

When $t = 1, s = 96$: $\frac{1}{2}a(1)^2 + v_0(1) + s_0 = 96 \Rightarrow \frac{1}{2}a + v_0 + s_0 = 96$

When $t = 2, s = 0$: $\frac{1}{2}a(2)^2 + v_0(2) + s_0 = 0 \Rightarrow 2a + 2v_0 + s_0 = 0$

Use Gaussian elimination to solve the system.

$$\begin{aligned} s_0 &= 160 \\ \tfrac{1}{2}a + v_0 + s_0 &= 96 \\ 2a + 2v_0 + s_0 &= 0 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 192 \\ 2a + 2v_0 + s_0 &= 0 \\ s_0 &= 160 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 192 \\ -2v_0 - 3s_0 &= -384 \qquad (-2)\text{ Eq. 1} + \text{Eq. 2} \\ s_0 &= 160 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 192 \\ v_0 + \tfrac{3}{2}s_0 &= 192 \qquad \left(-\tfrac{1}{2}\right)\text{ Eq. 2} \\ s_0 &= 160 \end{aligned}$$

$$\begin{aligned} s_0 = 160 &\Rightarrow s_0 = 160 \\ v_0 + \tfrac{3}{2}(160) = 192 &\Rightarrow v_0 = -48 \\ a + 2(-48) + 2(160) = 192 &\Rightarrow a = -32 \end{aligned}$$

The position equation is $s = \frac{1}{2}(-32)t^2 - 48t + 160$, or $s = -16t^2 - 48t + 160$.

(b) When $t = 1,\ s = 134$: $\frac{1}{2}a(1)^2 + v_0(1) + s_0 = 134 \Rightarrow a + 2v_0 + 2s_0 = 268$

When $t = 2,\ s = 86$: $\frac{1}{2}a(2)^2 + v_0(2) + s_0 = 86 \Rightarrow 2a + 2v_0 + s_0 = 86$

When $t = 3,\ s = 6$: $\frac{1}{2}a(3)^2 + v_0(3) + s_0 = 6 \Rightarrow 9a + 6v_0 + 2s_0 = 12$

Use Gaussian elimination to solve the system.

$$\begin{aligned} a + 2v_0 + 2s_0 &= 268 \\ 2a + 2v_0 + s_0 &= 86 \\ 9a + 6v_0 + 2s_0 &= 12 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 268 \\ -2v_0 - 3s_0 &= -450 \qquad (-2)\text{Eq.1} + \text{Eq.2} \\ -12v_0 - 16s_0 &= -2400 \qquad (-9)\text{Eq.1} + \text{Eq.3} \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 268 \\ -2v_0 - 3s_0 &= -450 \\ 3v_0 + 4s_0 &= 600 \qquad \left(-\tfrac{1}{4}\right)\text{Eq.3} \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 268 \\ -2v_0 - 3s_0 &= -450 \\ -s_0 &= -150 \qquad 3\text{Eq.2} + 2\text{Eq.3} \end{aligned}$$

$$\begin{aligned} -s_0 = -150 &\Rightarrow s_0 = 150 \\ -2v_0 - 3(150) = -450 &\Rightarrow v_0 = 0 \\ a + 2(0) + 2(150) = 268 &\Rightarrow a = -32 \end{aligned}$$

The position equation is $s = \frac{1}{2}(-32)t^2 + (0)t + 150$, or $s = -16t^2 + 150$.

(c) When $t = 1, s = 184$: $\frac{1}{2}a(1)^2 + v_0(1) + s_0 = 134 \Rightarrow a + 2v_0 + 2s_0 = 368$

When $t = 2, s = 116$: $\frac{1}{2}a(2)^2 + v_0(2) + s_0 = 116 \Rightarrow 2a + 2v_0 + s_0 = 116$

When $t = 3, s = 16$: $\frac{1}{2}a(3)^2 + v_0(3) + s_0 = 16 \Rightarrow 9a + 6v_0 + 2s_0 = 32$

Use Gaussian elimination to solve the system.

$$\begin{aligned} a + 2v_0 + 2s_0 &= 368 \\ 2a + 2v_0 + s_0 &= 116 \\ 9a + 6v_0 + 2s_0 &= 32 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 368 \\ -2v_0 - 3s_0 &= -620 \qquad (-2)\text{ Eq. 1} + \text{Eq. 2} \\ -12v_0 - 16s_0 &= -3280 \qquad (-9)\text{ Eq. 1} + \text{Eq. 3} \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 368 \\ v_0 + \tfrac{3}{2}s_0 &= 310 \qquad \left(-\tfrac{1}{2}\right)\text{ Eq. 2} \\ -12v_0 - 16s_0 &= -3280 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 368 \\ v_0 + \tfrac{3}{2}s_0 &= 310 \\ 2s_0 &= 440 \qquad 12\text{ Eq. 2} + \text{Eq. 3} \end{aligned}$$

$$\begin{aligned} 2s_0 = 440 &\Rightarrow s_0 = 220 \\ -2v_0 - 3(220) = -620 &\Rightarrow v_0 = -20 \\ a + 2(-20) + 2(220) = 368 &\Rightarrow a = -32 \end{aligned}$$

The position equation is $s = -\frac{1}{2}(-32)t^2 + (-20)t + 220$, or $s = -16t^2 - 20t + 220$.

72. Applying Kirchoff's first law to either junction produces

$I_1 + I_3 = I_2$ and applying the second law to the two paths produces

$R_1I_1 + R_2I_2 = 3I_1 + 4I_2 = 3$

$R_2I_2 + R_3I_3 = 4I_2 + 2I_3 = 2.$

Rearrange these equations, form the augmented matrix, and use Gauss-Jordan elimination.

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 3 & 4 & 0 & 3 \\ 0 & 4 & 2 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{5}{13} \\ 0 & 1 & 0 & \frac{6}{13} \\ 0 & 0 & 1 & \frac{1}{13} \end{bmatrix}$$

So, the solution is $I_1 = \frac{5}{13}$, $I_2 = \frac{6}{13}$, and $I_3 = \frac{1}{13}$.

Project Solutions for Chapter 1

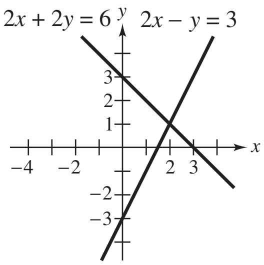
1 Graphing Linear Equations

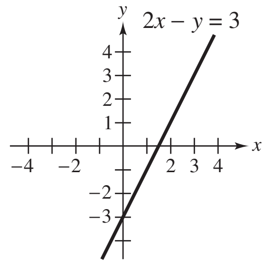
1. $\begin{bmatrix} 2 & -1 & 3 \\ a & b & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & b + \frac{1}{2}a & 6 - \frac{3}{2}a \end{bmatrix}$

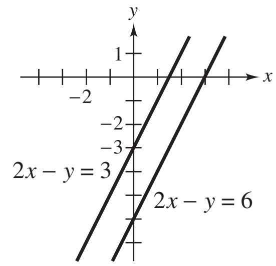
(a) Unique solution if $b + \frac{1}{2}a \neq 0$. For instance, $a = b = 2$.

(b) Infinite number of solutions if $b + \frac{1}{2}a = 6 - \frac{3}{2}a = 0 \Rightarrow a = 4$ and $b = -2$.

(c) No solution if $b + \frac{1}{2}a = 0$ and $6 - \frac{3}{2}a \neq 0 \Rightarrow a \neq 4$ and $b = -\frac{1}{2}a$. For instance, $a = 2, b = -1$.

(d) 





(a) $\begin{aligned} 2x - y &= 3 \\ 2x + 2y &= 6 \end{aligned}$ (b) $\begin{aligned} 2x - y &= 3 \\ 4x - 2y &= 6 \end{aligned}$ (c) $\begin{aligned} 2x - y &= 3 \\ 2x - y &= 6 \end{aligned}$

(The answers are not unique.)

2. (a) $\begin{aligned} x + y + z &= 0 \\ x + y + z &= 0 \\ x - y - z &= 0 \end{aligned}$ (b) $\begin{aligned} x + y + z &= 0 \\ y + z &= 1 \\ z &= 2 \end{aligned}$ (c) $\begin{aligned} x + y + z &= 0 \\ x + y + z &= 1 \\ x - y - z &= 0 \end{aligned}$

(The answers are not unique.)

There are other configurations, such as three mutually parallel planes or three planes that intersect pairwise in lines.

2 Underdetermined and Overdetermined Systems of Equations

1. Yes, $x + y = 2$ is a consistent underdetermined system.

2. Yes,

$$\begin{aligned} x + y &= 2 \\ 2x + 2y &= 4 \\ 3x + 3y &= 6 \end{aligned}$$

is a consistent, overdetermined system.

3. Yes,

$$\begin{aligned} x + y + z &= 1 \\ x + y + z &= 2 \end{aligned}$$

is an inconsistent underdetermined system.

4. Yes,

$$\begin{aligned} x + y &= 1 \\ x + y &= 2 \\ x + y &= 3 \end{aligned}$$

is an inconsistent underdetermined system.

5. In general, a linear system with more equations than variables would probably be inconsistent. Here is an intuitive reason: Each variable represents a degree of freedom, while each equation gives a condition that in general reduces number of degrees of freedom by one. If there are more equations (conditions) than variables (degrees of freedom), then there are too many conditions for the system to be consistent. So you expect such a system to be inconsistent in general. But, as Exercise 2 shows, this is not always true.

6. In general, a linear system with more variables than equations would probably be consistent. As in Exercise 5, the intuitive explanation is as follows. Each variable represents a degree of freedom, and each equation represents a condition that takes away one degree of freedom. If there are more variables than equations, in general, you would expect a solution. But, as Exercise 3 shows, this is not always true.